HYDRODYNAMICS AND HEAT TRANSFER FOR THE SINGLE-PHASE FLOW OF A COOLANT IN A POROUS ELECTRIC CABLE

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Transfer processes in a cryogenic coaxial electric cable with a porous inner tube are analyzed; the analysis reveals ways of intensifying the outflow of the heat generated when electric power is transmitted along the cable.

The creation of electric transmission lines cooled to a very low temperature (cryogenic lines) raises the problem of removing the heat generated in the cable in an efficient manner. Cryogenic lines are subdivided into the hyperconducting type, which make use of the sharp drop in the resistance of pure conductors with falling temperature, and the superconducting type, in which the phenomenon of superconductivity is exploited. In transmitting electrical power along lines of these or any other types heat evolution may occur. This is associated firstly with the ordinary Joule losses and secondly with hysteresis losses under ac conditions [1].

In order to ensure a rapid outflow of the heat generated in the cable for a small temperature drop, it is desirable to increase the area of the surface flushed by the coolant. This is achieved by the construction illustrated in Fig. 1, as described in [2]. Conductors in the form of wire braids are placed on both sides of a tube made from porous electrically insulating material II. The coolant is pumped along the inside of the porous tube and in the gap between the tube and the outer screen. Filtering through the porous tube in its travel through the system, the coolant eliminates the heat generated in the latter.

In this paper we shall present a theoretical analysis of the hydrodynamic and heat-transfer characteristics of this type of system for the case of a homogeneous coolant (without any phase transformations) flowing along it, gaseous helium in particular.

Heat flows into the system through the surface $r = R_3$ owing to the imperfection of the outer thermal insulation. We call the thermal flux density through this surface q_3. The wire braids on the two surfaces of the porous tube are fairly thin compared with the tube thickness. Hence the heat evolution which occurs in the wires when an electric current is passed through them may be expressed in the form of two surface heat sources concentrated on the surfaces $r = R_1$ and $r = R_2$. We denote the intensities of these sources by $\bar{q}_1$ and $\bar{q}_2$. We neglect the heat evolution associated with the dielectric losses in the material of the porous tube by comparison with that in the conductors. If necessary this neglected term may easily be incorporated in the problem about to be formulated.

As estimation of the Reynolds (Re) number for the flow of gaseous helium shows that, even for a comparatively low coolant velocity at the entrance into the cable (~0.4 m/sec), the Re number is more than an order of magnitude greater than the critical number Re_{cr} for flows along infinitely long, impermeable tubes. This shows that the helium flow in the initial section of the inner tube is, as a rule, turbulent. As the coolant permeates into the coaxial gap, its velocity in the inner tube diminishes on passing down the flow together with the Re number, and the latter may fall below Re_{cr}. In this case the flow becomes

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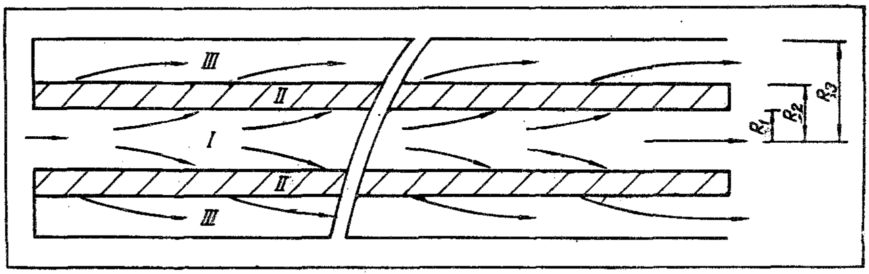

Fig. 1. Coaxial cable (schematic): I) Inner region; II) porous electrically-insulating tube; tube; III) coaxial gap.

laminar, and will subsequently remain so all along the tube.

If the coaxial gap is closed at the initial point of the cable, the opposite will occur. In the initial section of the gap the helium velocity will be low and the flow laminar. As coolant penetrates into the gap the velocity of the gas will increase down the flow and the flow may become turbulent.

1. Let us first present the case for a laminar flow of coolant. Since the length of the cable L is much greater than the transverse dimensions of the channel (R_1 or R_3-R_2), the flow of coolant in regions I and III may be described by boundary-layer equations. In a cylindrical coordinate system these have the form:

$$\frac{\partial r\rho v_x}{\partial x} + \frac{\partial r\rho v_r}{\partial r} = 0; \tag{1}$$

$$\rho v_x \frac{\partial v_x}{\partial x} + \rho v_r \frac{\partial v_x}{\partial r} = -\frac{\partial P}{\partial x} + \frac{\eta}{r} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial v_x}{\partial r}\right); \tag{2}$$

$$\frac{\partial P}{\partial r} = 0; \tag{3}$$

$$\frac{\partial r\rho v_x h}{\partial x} + \frac{\partial r\rho v_r h}{\partial r} = \frac{\partial}{\partial r}\left(r\lambda \frac{\partial T}{\partial r}\right). \tag{4}$$

Before writing down the transfer equations in the porous wall of the tube (region II in Fig. 1), we should make some comments regarding the character of the transfer processes taking place in this region. First of all, owing to the small thickness of the tube wall as compared with its length, we may neglect filtration and heat conduction in the longitudinal direction. Secondly, we shall assume local equality between the temperatures of the filtering gas and the porous matrix. Finally, we shall describe filtration on the basis of the Darcy law. On these assumptions the transfer processes in the tube walls will be described by the following system of equations:

$$\frac{\partial r\rho v_r}{\partial r} = 0; \quad v_x = 0; \tag{5}$$

$$\frac{\partial P}{\partial r} = -\frac{\eta}{K} v_r; \tag{6}$$

$$\frac{\partial}{\partial r} r\rho v_r h = \frac{\partial}{\partial r}\left(r\lambda' \frac{\partial T}{\partial r}\right). \tag{7}$$

The transfer equations, written out separately for each of the regions in question, have to be supplemented by the equation of state

$$P = P(\rho, T). \tag{8}$$

For a pressure close to atmospheric, and over a temperature range of the order of a few degrees, the specific heat of gaseous helium varies very slowly. We may therefore put $h = c_p T$.

In formulating the boundary conditions and the conditions of conjugation for the foregoing equations, we shall distinguish the corresponding quantities in each of the regions by the indices 1, 2, 3. Owing to the effects of adhesion, the v_x velocity component should vanish at the tube walls:

$$v_{x_1}|_{r=R_1} = v_{x_2}|_{r=R_2} = v_{x_3}|_{r=R_3} = 0. \tag{9}$$

At the surface $r = R_1$ and $r = R_2$ we must satisfy the mass-flow continuity and energy-conservation conditions:

$$(\rho_1 v_{r_1})_{r=R_1} = (\rho_2 v_{r_2})_{r=R_1}; \tag{10}$$

$$(\rho_2 v_{r_2})_{r=R_2} = (\rho_3 v_{r_3})_{r=R_2}; \tag{11}$$

$$\left[\rho_2 v_{r_2} h_2 - \lambda' \frac{\partial T_2}{\partial r}\right]_{r=R_1} - \left[\rho_1 v_{r_1} h_1 - \lambda \frac{\partial T_1}{\partial r}\right]_{r=R} = \bar{q}_1; \tag{12}$$

$$\left[\rho_3 v_r h_3 - \lambda \frac{\partial T_3}{\partial r}\right]_{r=R_2} - \left[\rho_2 v_{r_2} h_2 - \lambda' \frac{\partial T_2}{\partial r}\right]_{r=R_2} = \bar{q}_2. \tag{13}$$

In addition to this, the surfaces $r = R_1$ and $r = R_2$ should be characterized by continuity of the helium temperature

$$T_1|_{r=R_1} = T_2|_{r=R_1}; \quad T_3|_{r=R_2} = T_2|_{r=R_2} \tag{14}$$

and pressure

$$P_1|_{r=R_1} = P_2|_{r=R_1}; \quad P_3|_{r=R_2} = P_2|_{r=R_2}. \tag{15}$$

A flux q_3 passes through the outer shell. Hence

$$-\lambda \frac{\partial T}{\partial r}\Big|_{r=R_3} = q_3. \tag{16}$$

Supplementary conditions apply at the entry into and exit from the cable. We shall discuss these in detail below.

2. The turbulent flow of the coolant may also be described by the boundary-layer equations, but it is very troublesome to calculate the flow on the basis of any semiempirical theory of the turbulent boundary layer. However, we may confine ourselves to a more approximate calculation, that of finding the flow characteristics averaged over the cross section of the channel.

Owing to the intense agitation taking place, in the case of turbulent flow such quantities as ρ and T should depend only slightly on the transverse coordinate r. Bearing this in mind, and averaging the equations of the turbulent boundary layer over the cross section of the inner tube, we arrive at the following system of equations:

$$\frac{d}{dx}(\rho_1 \bar{v}_{x_1}) = -\frac{2(\rho_1 v_{r_1})_{r=R_1}}{R_1}; \tag{17}$$

$$\frac{d}{dx}[P_1 + \beta\rho_1 \bar{v}_{x_1}^2] = -\frac{2\tau_\omega}{R_1}; \tag{18}$$

$$\frac{d}{dx}(\rho_1\bar{v}_{x_1}h_1)+\frac{2}{R_1}(\rho v_{r_1}h_1)_{r=R_1}=\frac{2}{R_1}q_{r_1}|_{r=R_1}, \tag{19}$$

where

$$\bar{v}_{x_1}=\frac{2}{R_1^2}\int_0^{R_1}v_{x_1}rdr;\quad \beta=\frac{\overline{v_{x_1}^2}}{\bar{v}_{x_1}^2}. \tag{20}$$

The system of equations (8), (17)-(20) is not closed. In order to solve this system we need information as to the momentum flow coefficient β and the frictional force per unit lateral surface area τ_ω; these must be obtained experimentally.

An analogous averaging procedure leads to the following system of equations describing the turbulent flow of the gas in the coaxial gap:

$$\frac{d}{dx}[\rho_3\bar{v}_{x_3}]=\frac{2R_2}{R_3^2-R_2^2}(\rho_3 v_{r_3})_{r=R_2}; \tag{21}$$

$$\frac{d}{dx}[P_3+\beta\rho_3\bar{v}_{x_3}^2]=-\frac{2}{R_3^2-R_2^2}(R_3\tau_\omega|_{r=R_3}+R_2\tau_\omega|_{r=R_2}); \tag{22}$$

$$\frac{d}{dx}[\rho_3\bar{v}_{x_3}h_3]=\frac{2}{R_3^2-R_2^2}[R_2(\rho_3 v_{r_3}h_3+q_{r_3})_{r=R_2}-R_3 q_3|_{r=R_3}], \tag{23}$$

$$\bar{v}_{x_3}=\frac{2}{R_3^2-R_2^2}\int_{R_2}^{R_3}v_{x_3}rdr;\quad \beta=\frac{\overline{v_{x_3}^2}}{\bar{v}_{x_3}^2}. \tag{24}$$

Experimental data regarding the coefficients β and $\zeta = 8\tau\omega/\rho\bar{v}_x^2$ for flows along tubes with injection and suction are presented in [3, 4]. These amount to the following: 1) Starting at a certain distance from the entry into the permeable section ($\approx 8R$), β ceases to depend on x; for the case of suction β is close to unity; 2) in the case of injection the coefficient of friction ζ is very similar to the value which it assumes in an impermeable tube. In the case of suction the friction depends on the suction coefficient $K_1 = v_r|_{r=R_1}/\bar{v}_x$ and is given by the equation $\zeta = 17.5\ K_1$.

Integration of Eqs. (5)-(7), with due allowance for conditions (10)-(15), yields the following expressions for the quantities on the right-hand sides of Eqs. (17)-(19) and (21)-(23) for $(\rho_1 v_{r_1})_{r=R_1}$, $(\rho_3 v_{r_3})_{r=R_2}$, $q_{r_1}|_{r=R_1}$, $q_{r_3}|_{r=R_2}$:

$$(\rho_1 v_{r_1})_{r=R_1}=c/R_1;\quad (\rho_3 v_{r_3})_{r=R_2}=c/R_2; \tag{25}$$

$$q_{r_1}|_{r=R_1}=\frac{G}{R_1}-\frac{c}{R_1}c_{p_1}T_1-\bar{q}_1; \tag{26}$$

$$q_{r_3}|_{r=R_2}=\frac{G}{R_2}-\frac{c}{R_2}c_{p_3}T_3+\bar{q}_2, \tag{27}$$

where

$$c=\frac{P_1-P_3}{\eta_2\ln\frac{R_2}{R_1}}\rho_2 K; \tag{28}$$

$$G=cc_{p_2}\left[T_1\left(\frac{R_2}{R_1}\right)^{\frac{cc_{p_2}}{\lambda'}}-T_2\right]\Big/\left(\frac{R_1}{R_2}\right)^{\frac{cc_{p_2}}{\lambda'}}-1. \tag{29}$$

3. Filtration through the porous barrier leads to a fall in the coolant velocity (on passing down the flow) in the region from which it is subject to suction; ultimately the flow becomes laminar (if it were initially turbulent). The transition from turbulent to laminar flow takes place over a finite region, and it is very difficult to calculate the flow over this. At the same time, in order to calculate the laminar flow following this region we have to know the velocity profile at its exit point. It should be added that Re_{cr} is also unknown for flows along tubes with injection and suction. For these reasons in the present investigation we decided to base our calculations of the flow of coolant along the whole cable on Eqs. (17)-(19), (21)-(23), (8), (25)-(29). This simplification should not have any serious effect on the results of the calculations, since for sufficiently high velocities of the coolant the regions of laminar gas flow should make up only a very small part of the length of the whole section of cable.

In order to solve the system of ordinary differential equations (17)-(19) and (21)-(23), we have to supply six boundary conditions. Some of these may be given at the entrance into the part under consideration and some at the outlet; where and what conditions should be specified depends on the particular physical situation. For the equations of energy (19) and (23), the temperature of the coolant injected into the inner tube and the annular gap should be given at the entrance into the cable. The conditions for the hydrodynamic equations depend on how the pumping of the coolant through the cable is organized. Specified pressures $P_1|_{x=0}$, $P_3|_{x=0}$, $P_1|_{x=L}$, $P_3|_{x=L}$, many be created at the entrance and outlet of the cable. The difference between these then determines the rate of flow of the coolant. In this case we shall have an end problem. There may also be other ways of expressing the boundary conditions. In particular, when all the conditions are specified at the entrance into the cable, we have an initial problem. Our present numerical calculations were carried out for this latter case.

First of all, we studied the hydrodynamic flow pattern for the isothermal case without the energy equations. The coaxial gap was assumed to be covered at the entrance into the cable, i.e., $\bar{v}x_3|_{x=0}=0$. In addition to this, the total flow of coolant $(\rho_1\bar{v}x_1)_{x=0}$ and its pressure $P_1|_{x=0}$ were specified at the entrance into the porous tube. The fourth condition (the pressure at the initial point of the coaxial gap) was varied. Depending on the value of this pressure, different flow patterns were obtained. The equation of state for helium was taken from [5].

It should be noted that, from the point of view of screening a superconductor against the thermal flux q_3 arriving from outside, the most desirable mode of flow is that in which filtration of the gas from the inner tube into the coaxial gap through the porous wall takes place along the whole cable. The pressure distribution along the tube and the coaxial gap corresponding to this case is shown in Fig. 2a. For specified $\bar{v}x_3|_{x=0} = 0$, $(\rho_1\bar{v}x_1)_{x=0}$, $P_1|_{x=0}$ K and L, a flow of this kind is realised, not for all $P_3|_{x=0}$, but only for values lying in the range (P', P"). If $P_3|_{x=0}$, is made less than P'_3, the flow pattern changes. As before, there will be suction of the fluid from the inner tube into the coaxial gap, but starting from a certain point x_0 (for $x > x_0$) reverse flow will develop in the inner tube, accompanied by a rise in pressure. With increasing x there will be a very rapid swing in the solution. The point P' corresponds to the case in which $x_0 = L$.

In addition to (P', P") there is yet another range of values of the pressure $P_3|_{x=0} >$ P", namely, (P', P'''), in which there are no reverse flows. In this case for a certain x_0' there will be a change in the direction of filtration, as in Fig. 2b. For $x < x_0'$, as before suction will take place from the inner tube into the coaxial gap, while for $x > x_0'$ it will proceed in the opposite direction. The case $P_3|_{x=0} = P''$ corresponds to $x_0' = L$.

Finally if $P_3|_{x=0} > P'''$, reverse flow occurs in the coaxial gap for $x > x_0''$. With increasing x there is also a rapid swing in this solution. The condition $P_3|_{x=0} = P'''$ corresponds to the case $x_0'' = L$.

The range of pressures $P_3|_{x=0}$ for which no reverse flows nor rapid swings in the solution occur along the whole length L of the cable is the range (P', P'''). The extent of this depends on the permeability of the porous tube wall K and length of the cable L. A reduction in these quantities expands the range (P', P") and an increase contracts it.

For a fixed permeability, an increase in the length of the cable leads to a reduction in the range (P', P'''). The value of L for which the range (P', P''') degenerates into a point is the limiting length of the cable for which stable flow of the coolant remains possible.

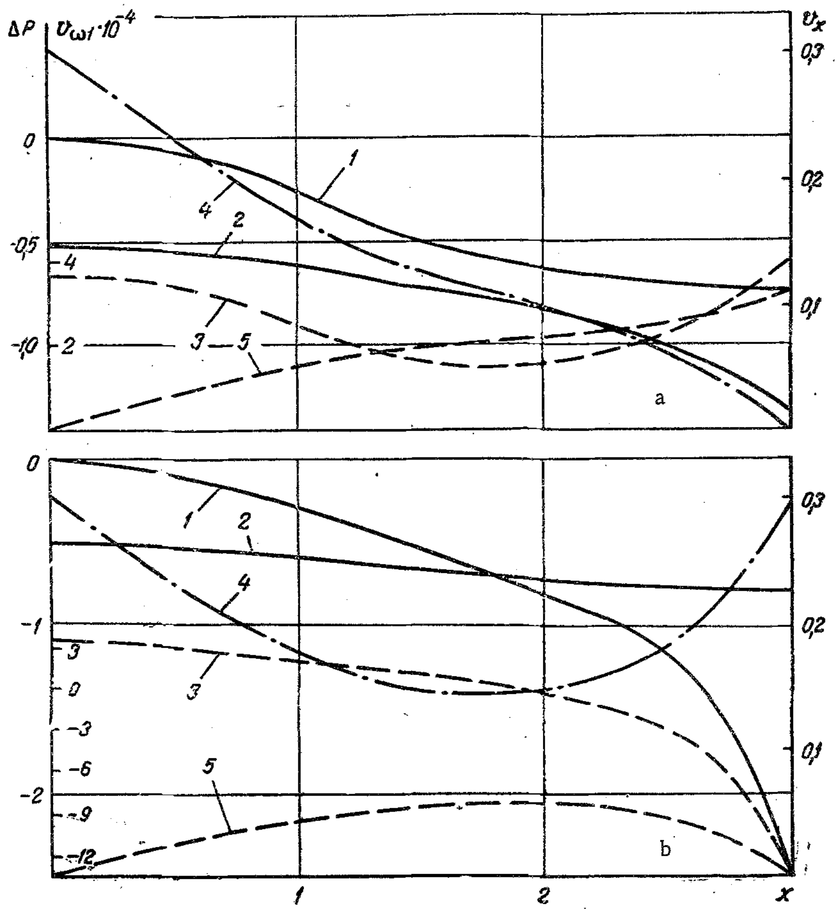

Fig. 2. Distribution of the hydrodynamic characteristics along the cable for K = 3.5 Darcy for $P_3|_{x=0} = P'$ a) and $P_3|_{x=0} = P'''$ b): 1) $P_1 - P_1|_{x=0}$; 2) $P_3 - P_1|_{x=0}$; 3) $v_{\omega 1} = v_{r1}|_{r=R_1}$; 4) v_{x1}; 5) v_{x3}. ΔP, N/m^2; $v_{\omega 1}$, m/sec; v_x, m/sec; x, m.

Thus the stability of flow in the cable under consideration largely depends on the permeability of the tube wall K, increasing as K falls.

We calculated the flow of gaseous helium for the following parameters $R_1 = 0.005$ m, $R_2 = 0.01$ m; $R_3 = 0.013$ m, $v_{x1}|_{x=0} = 0.3$ m/sec, $\rho|_{x=0} = 14$ kg/m^3; T = 5°K; $P_1|_{x=0} = 113{,}966$ N/m^2.

For a permeability of 35 Darcy the limiting length of the cable was 2 m. The range (P', P''') for a cable 3 m long and a permeability K = 3.5 was 0.02 N/m^2, and for a permeability 0.35 ≈ 2.2 N/m^2. The results of the calculations for the two extreme cases of $P_3|_{x=0} = P'$ and $P_3|_{x=0} = P'''$ with K = 3.5 are presented in Fig. 2a and b.

These calculations lead to the conclusion that the technical realization of optimum coolant flow in cables over 10-20 m long is very problematical. This is because, for long cables, the filtration velocity of the coolant passing from the inner tube to the coaxial gap (proportional to 1/L) is too low. In this case the pressure drop at the porous wall $P_1 - P_3 \ll P_1|_{x=0} - P_1|_{x=L}$. The perturbations arising in the flow are comparable with $P_1 - P_3$; they distort the filtration of the coolant very severely, and this leads to instability of the flow.

NOTATION

x, r, axes of the cylindrical coordinate system; P, pressure; ρ, density; v, velocity; η, dynamic viscosity, λ, thermal conductivity; h, specific enthalpy of the gas; T, temperature; K, permeability; λ', thermal conductivity of the porous matrix, the pores being filled with gas; c_p, specific heat of the gas at constant pressure; q, energy flux.

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